DYNAMIC BEHAVIOR STUDY OF PERFORATED STRING SYSTEM UNDER IMPACT LOAD

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ABSTRACT
In order to assess the safety of perforated string and test equipment under impact load, a dynamic model of the perforated string-shock absorber system is developed using micro-element method. In the model, the perforated string’s weight, actual length and fluid friction are taken into account. The corresponding initial conditions, boundary conditions and difference scheme of the model are given. Using the model, the dynamic behavior of perforated string system in a drilling platform in South China Sea is investigated, based on which the value range of perforated string length, shock absorber stiffness coefficient, shock absorber damping coefficient of shock absorber are proposed. The results of study can provide effective theoretical support for optimal design of perforated string and developing new type of shock absorber.

INTRODUCTION
During the deep-water oil&gas testing, instantaneous impact force from perforating operation can cause the strong longitudinal shock vibration of perforated string, huge impact load to the packer and great influence on testing equipment (see Figure1). In order to protect the testing string system from damage caused by the great instant impact load [1-4] and optimize the construction parameters for perforated string and shock absorber, it is very important to develop an effective dynamic model to investigate the dynamic behaviors of the perforated string system (PS) under perforating impact load [5-7].
Commercial software, such as ABAQUS, ANSYS and ADINA [8-10] are most frequently used to simulate the structure dynamic behavior. For example, recently ABAQUS has been used by WU [8] to study the influence of impact load on the string deformation. It is a good choice for professionals to use mature software based on finite element method (FEM) to make a qualitative analysis on the dynamic problem. However, it needs special computational mechanical knowledge to master such software, or else it is difficult to make right judgment to the calculation results. Moreover, in view of the different testing conditions, different analysis models need to be established. This will bring much inconvenience to the designers of oil equipment’s.

There are also some researchers [11-15] trying to develop dynamic models of the problem based on the principle of vibration mechanics. But in these models, neither the long-thin perforated string is regarded as infinitely long elastic
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pipe, nor is the deformation of the perforated string ignored with the perforated string considered as a rigid boundary of shock absorber. According to these works, the mechanical models of the long-thin perforated string system may be simplified excessively and are great different with the practice structure. In addition, in these models, other factors such as fluid pressure and fluid friction force acting on the perforated string cannot be taken into account [16].

At present, the models which can be used to effectively study the dynamic behaviors of the perforated string system under perforating impact load are still very lacking. In view of this, the first purpose of this paper is to develop a dynamics model in which the perforated string is viewed as an elastic pipe with finite length and fluid friction acting on the perforated string can be taken into account. The second purpose is to use the model to study the influence of perforated string length, shock absorber stiffness coefficient, shock absorber damping coefficient and quality of shock absorber on the dynamic behaviors of the perforated string system.

DYNAMIC MODELS

According to the structure and force analysis, the mechanic model of a perforated string-shock absorber system is shown in figure 2(a) [16]. The upper long-thin elastic pipe represents the perforated string, the lower quality-spring-damper system represents the shock absorber and the fixed constraint on the upper bottom reflects the fixed action of the packer to the perforated string. The lower bottom of the shock absorber is subjected to the perforated impact load.

\[
\begin{align*}
\rho A \frac{\partial^2 u_1(x,t)}{\partial t^2} &+ \frac{\partial u_1(x,t)}{\partial t} \frac{\partial}{\partial x} \left( EA \frac{\partial u_1(x,t)}{\partial x} \right) dx \\
\frac{\partial}{\partial x} \left( EA \frac{\partial u_1(x,t)}{\partial x} \right) dx &+ \rho Ag dx \\
\end{align*}
\]
In order to derive the dynamic model of the system, two coordinate axes, whose origins are respectively at the down bottom and quality point, are set up (figure 2(a)). Micro-element method and D'alembert principle are respectively used to establish the vibration equations of the long-thin elastic pipe and the quality-spring-damper system.

Using the micro-element method, an arbitrary micro section of the long-thin elastic pipe is cut off hypothetically and force analysis are made (figure 2(b)). In the figure 2(b), the following four physical quantities respectively represent the inertial force, the axial force, viscous damping force and gravity on the infinitesimal section.

\[
\rho A \frac{\partial^2 u_i(x,t)}{\partial t^2} dx + EA \frac{\partial^2 u_i(x,t)}{\partial x^2} dx + v \frac{\partial u_i(x,t)}{\partial t} dx + \rho Ag dx
\]

Where \(E, \rho\) and \(A\) respectively represent the elastic modulus, material density and cross sectional area of the perforated string. \(v\) is the damping coefficient of the internal and external liquid of the perforated string. \(g\) is the gravity acceleration. \(u_i(x,t)\) is the displacement of cross section whose position is \(x\) from the coordinate origin of perforated string at time \(t\).

According to the force equilibrium conditions of the infinitesimal section, the vibration partial differential equation of the perforated string is obtained

\[
\rho A \frac{\partial^2 u_i(x,t)}{\partial t^2} dx - EA \frac{\partial^2 u_i(x,t)}{\partial x^2} dx + v \frac{\partial u_i(x,t)}{\partial t} dx = \rho Ag dx
\]

With both sides divided by \(\rho Adx\), the above equation can be transformed into the following form.

\[
\frac{\partial^2 u_i(x,t)}{\partial t^2} - a^2 \frac{\partial^2 u_i(x,t)}{\partial x^2} + v \frac{\partial u_i(x,t)}{\partial t} = g_0
\]

Where

\[a = \sqrt{\frac{E}{\rho}}\] is the wave propagation speed in perforated string,

\(g_0\) is a constant that represents the gravity of pipe section with unite length \(\rho Ag\).
\[ v = \frac{12\pi\mu}{\rho A} \left( \frac{D_r}{D_i - D_r} \right) \left[ (0.20 + 0.39 \frac{D_r}{D_i}) + \frac{2.197 \times 10^4}{24} \left( \frac{D_c}{D_i} - 0.3810 \right)^{2.57} \frac{D_c^2 - D_r^2}{LD_r} \right] \]

Where

\( \mu \) is the dynamic viscosity of the internal and external liquid of the perforated string, Pa·s,

\( D_c \) is the outer diameter of the perforated string, m,

\( D_i \) is the inner diameter of the perforated string, m,

\( L \) is the length of the perforated string, m,

\( D_r \) is the inner diameter of the wellbore.

The vibration differential equation of the shock absorber system can be written as follow according to D’Alembert principle [18]

\[
m \frac{d^2u_2(t)}{dt^2} + c \frac{du_2(t)}{dt} + ku_2(t) = P(t) \tag{2}
\]

Where \( m \), \( k \) and \( c \) respectively represent the quality, stiffness and damping coefficient of the shock absorber, \( u_2(t) \) is the displacement of the shock absorber at time \( t \), \( P(t) \) represents the impact load acting on the shock absorber at time \( t \).

It is noted that the location of the dangling end, namely the down bottom of the perforated string changes with the deformation of the elastic string. Therefore the equation (2) can be adjusted as the following form in order to consider the location change of the dangling end.

\[
m \frac{d^2u_2(t)}{dt^2} + c \frac{d}{dt} \left[ u_2(t) + u_1(0,t) \right] + k \left[ u_2(t) + u_1(0,t) \right] = P(t) \tag{3}
\]

The equation (3) can be also expressed by the following formula
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\[ P(t) - F_s(t) - F_d(t) = m\ddot{u}_2(t) \]  \hspace{1cm} (4)

Where \( F_s(t) \) and \( F_d(t) \) respectively represent the spring force and the damping force and

\[ F_s(t) = k\left[ u_2(t) + u_1(0,t) \right], \quad F_d(t) = \frac{d}{dt}\left[ u_2(t) + u_1(0,t) \right]. \]

According to the force continuity condition on the joint between the perforated string and the shock absorber

\[ EA \frac{\partial u_1(x,t)}{\partial x} \bigg|_{x=0} = F_s(t) + F_d(t) \]  \hspace{1cm} (5)

Formula (5) can be further written as

\[ EA \frac{\partial u_1(x,t)}{\partial x} \bigg|_{x=0} = c \frac{d}{dt}\left[ u_2(t) + u_1(0,t) \right] + k\left[ u_2(t) + u_1(0,t) \right] \]  \hspace{1cm} (6)

Since the upper bottom of the perforated string is fixed, the following displacement boundary can be given

\[ u_1(L,t) = 0 \]  \hspace{1cm} (7)

It can be found from the above analysis that equations (1)(3)(6)(7) describe together the dynamic model of the perforated string system under impact load namely

\[
\begin{align*}
\frac{\partial^2 u_1(x,t)}{\partial t^2} - a^2 \frac{\partial^2 u_1(x,t)}{\partial x^2} + v \frac{\partial u_1(x,t)}{\partial t} &= g_0 \\
m \frac{d^2 u_2(t)}{dt^2} + c \frac{d}{dt}\left[ u_2(t) + u_1(0,t) \right] + k\left[ u_2(t) + u_1(0,t) \right] &= P(t) \\
u_1(L,t) &= 0 \\
EA \frac{\partial u_1(x,t)}{\partial x} \bigg|_{x=0} &= c \frac{d}{dt}\left[ u_2(t) + u_1(0,t) \right] + k\left[ u_2(t) + u_1(0,t) \right]
\end{align*}
\]  \hspace{1cm} (8)

DIFFERENCE SCHEME
Newton difference technique is used to solve the equations (8). The perforated string is discretized uniformly with \((N+1)\) nodes along its axis direction whose numbers are \(i = 0, 1, \cdots, N\) from top to bottom. The time of the vibration response is divided into \(K+1\) spans whose sizes are \(\Delta t\) and numbers are \(j = 0, 1, \cdots, K\).

According to the Newton forward difference formula [19], the movement speed of the \(i\)th point at the \(j\)th and the \((j-1)\)th moment can be respectively expressed by the following two formulas approximately.

\[
\left(\frac{\partial u}{\partial t}\right)_{i,j} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \quad (9a)
\]

\[
\left(\frac{\partial u}{\partial t}\right)_{i,j-1} = \frac{u_{i,j} - u_{i,j-1}}{\Delta t} \quad (9b)
\]

Where \(u_{i,j}\) represents the displacement of the \(i\)th point at the \(j\)th moment.

Similarly, the second-order partial differential items in equation (8) can also be written approximately as follows.

\[
\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \frac{\left(\frac{\partial u}{\partial t}\right)_{i,j} - \left(\frac{\partial u}{\partial t}\right)_{i,j-1}}{\Delta t} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta t^2} \quad (10a)
\]

\[
\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} \quad (10b)
\]

Substituting Eq. (9) and (10) into the system equations (8a) and (8b) yields the following differential scheme.
At the \((j+1)\)th moment, the displacements of all the discrete points except for the down endpoint of the string can be calculated using formula (11a) and (8c) based on the displacements at \(j\)th and \((j-1)\)th moment. Equation (7b) contains the displacement of the down endpoint \((N)\) of the string and the displacement of the shock absorber \((N+1)\). Therefore, in order to obtain the displacements of the discrete point \(N\) and \(N+1\) at the \((j+1)\)th moment, another complementary equation should be given. In order to do this, Newton backward difference formula is used to discretize equation (8d)

\[
\left( \frac{m}{\Delta t^2} + \frac{c}{2\Delta t} \right) u_{N+1,j+1} + \frac{c}{\Delta t} u_{N,j+1} = P_j - \left( k - \frac{2m}{\Delta t^2} \right) u_{N+1,j} - \left( \frac{m}{\Delta t^2} - \frac{c}{2\Delta t} \right) u_{N+1,j-1} + \left( \frac{c}{\Delta t} - k \right) u_{N,j} \quad 1 \leq j \leq K \quad (11b)
\]

Where \(\Delta x\) is the length of differential grid closed to the down endpoint, \((P_p)_{j+1}\) is the forces acting on the endpoint of perforated string at the moment \(j+1\), which is caused by spring and damping between perforated string and shock absorber. Therefore Eq. (12) can be further written as follows by replacing the \((P_p)_{j+1}\) with the spring force and the damping force which are calculated based on the relative displacements between the down endpoint of perforated string and the shock absorber.

\[
\left( \frac{2\Delta k}{3EA} - \frac{\Delta c}{3EA\Delta t} \right) u_{N+1,j+1} + \left( 1 - \frac{2\Delta k}{3EA} - \frac{2\Delta c}{3EA\Delta t} \right) u_{N,j+1} = \frac{4}{3} u_{N-1,j+1} - \frac{1}{3} u_{N-2,j+1} + \frac{2(P_p)_{j+1} \Delta x}{3EA} \quad 1 \leq j \leq K \quad (13)
\]
Solving the equation (11b) and the equation (13) simultaneously yields the displacements of the down endpoint (N) of perforated string and the shock absorber (N+1) at the moment \( j + 1 \).

In order to solve the differential equations (8c) (11) (13) simultaneously, the difference scheme of the initial conditions must be given. It is assumed that in the initial state, the displacements of every differential point including shock absorber are zero. Therefore the following equation is set up

\[
u_{i,0} = u_{i,1} = u_{0,0} \quad (14)
\]

It can be found that in the equation (11a) the displacement iteration calculation begins from the second moment. Therefore the displacements at the first moment need to be determined. It is assumed that at the moment \( t = 0 \),

\[
\frac{\partial u_{i,1}}{\partial t} = 0 \text{ which can be expressed as follows by centre differential scheme}
\]

\[
\frac{\partial u_{i,1}}{\partial t} = \frac{u_{i,2} - u_{i,0}}{2\Delta t} = 0 \quad (15)
\]

According to the formula (15), the relation \( u_{i,0} = u_{i,2} \) is obtained. Substituting the relation into the equation (11a) and taking \( j = 1 \) yields the following equation

\[
u_{i,2} = \frac{e(u_{i+1,1} + u_{i-1,1}) + (2 - v\Delta t - 2e)u_{i,1} + g\Delta t^2}{1 + v\Delta t} \quad (16)
\]

Where \( e = \frac{a^2\Delta t^2}{\Delta x^2} \).

The similar deduce process can be used to determine the \( u_{N+1,2} \) that represents the displacement of the shock absorber at the second moment.
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\[
\left( \frac{m}{\Delta t^2} + \frac{c}{2\Delta t} \right) u_{N+1,2} + \frac{c}{\Delta t} u_{N,2} \\
= P_1 - \left( k - \frac{2m}{\Delta t^2} \right) u_{N+1,1} - \left( \frac{m}{\Delta t^2} - \frac{c}{2\Delta t} \right) u_{N+1,0} + \left( \frac{c}{\Delta t} - k \right) u_{N,1}
\]

(17)

Using the equation (11), (13), (16), (17) in combination with the initial state at time zero, the displacements response as well as axial force response of the perforated string system can be studied.

**DYNAMIC BEHAVIOR OF PS SYSTEM UNDER IMPACT LOAD**

**Basic parameter of PS system**

Using the dynamic model, the dynamic behavior of a practical drilling perforated string system in the South China Sea is investigated. In the calculation, the total computation time is 80s with the time step span \( \Delta t = 0.005 \). The basic system parameters are shown in table 1.

<table>
<thead>
<tr>
<th>Material of the perforated string</th>
<th>Young’s modulus of the perforated string</th>
<th>Shear modulus of the perforated string</th>
<th>Outer diameter of the perforated string</th>
</tr>
</thead>
<tbody>
<tr>
<td>73mm UP TBG, grade N80 steel</td>
<td>206GPa</td>
<td>79.4GPa</td>
<td>100mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inner diameter of the perforated string</th>
<th>Inner diameter of the wellbore</th>
<th>Dynamic viscosity of the internal and external liquid of the perforated string</th>
<th>Impact load peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>50mm</td>
<td>120mm</td>
<td>0.042Pa·s</td>
<td>8.0×10^5N</td>
</tr>
</tbody>
</table>

The decay curve of perforating impact load shown in figure 3 is used where the pressure value at stable stage and peak pressure are respectively 59.539 MPa and 140.069 MPa. Typical values of decay curve of the perforating impact load are listed in table 2.
Influence of perforated string length on PS system

The dynamic responses of the PS system with different perforated string length are shown in Figure 4. In figure 4(a), it can be found that the reaction force on the fixed end section of perforated string decreases gradually with the increase of perforated string length. The main reason is that the system energy is dissipated by shock absorber damping. In figure 4(b), the displacement-time curve of the below endpoint of perforated string shows that the longer the perforated string is, the larger of the amplitude. This is because each node of the perforated string has certain displacement, the longer the string, the greater the accumulated value of displacement.

In figure 4(c) and (d), with the decreasing of perforated string length, there are no significant change of shock absorber displacement and velocity. But the length decrease can lead to the increase of reaction force on the fixed end of perforated string. In this case, the instrument attached to the fixed end is more likely to be damaged. On the contrary, if the perforated string length increases, the reaction force on fixed end of the PS would reduce, whereas the amplitude of endpoint of the PS would increase. Therefore the PS length should be selected in a reasonable range. Through
comprehensive comparison in figure 4(a), (b), (c), (d) and (e), it can be found that the dynamic response of the PS system can be controlled in a reasonable range when the range of perforated length is selected between 30～40 m.

**Figure. 4 Dynamic response time-history curve of PS system with different perforated length**

**Influence of Shock Absorber Stiffness Coefficient on PS system**
Dynamic responses of the PS system with different stiffness coefficient of shock absorber are shown in figure 5. It is shown in the figure that with the increase of stiffness coefficient, the reaction force on fixed end section, the displacement of endpoint and the velocity of shock absorber decrease. Therefore increasing shock absorber stiffness coefficient is beneficial to the safety of the equipment above perforated string and perforated string. But increasing the shock absorber stiffness coefficient can bring difficulties to the shock absorber design. The value of shock absorber stiffness coefficient should be selected in a reasonable range. In this paper, the study found that the dynamic response of the PS system can be controlled in a reasonable range when the range of shock absorber stiffness coefficient is selected as 180～210 KN/m.
Influence of Shock Absorber Damping Coefficient to PS System

Dynamic responses of the PS system with different shock absorber damping coefficient are shown in figure 6. It is shown that the reaction force on fixed end of perforated string and the displacement amplitude of the endpoint of perforated string obviously increase with the increase of shock absorber damping coefficient. Therefore the increase of damping coefficient may result in the failure of equipment on perforated string. The figure 6(c) and (d) show that the displacement and velocity decrease with the increase of damping coefficient, so as to reduce the amplitude of the tubing string at the bottom of shock absorber and perforated gun. Then, the helical buckling of tubing string at the bottom of shock absorber is reduced. Therefore, the value of shock absorber damping coefficient in a reasonable range is necessary. Through the analysis and comparison in figure 6(a), (b), (c), (d) and (e), the reasonable range of shock absorber damping coefficient is $5 \sim 15 \text{KN} \cdot \text{s/m}$. 

Figure. 5 PS system dynamic response time history curve of a cross-section with different shock absorber stiffness coefficient.

(a) Fixed end section force of perforated string
(b) Displacement of endpoint of perforated string
(c) Displacement of shock absorber
(d) Velocity of shock absorber
CONCLUSIONS

In this paper, a vibration model of the PS system and its differential scheme are presented in detail. Using the model, the dynamic behavior of perforated string system in a drilling platform in South China Sea is investigated. The following conclusions are obtained.

1) Compared with the existing models, the dynamic model in this paper is more close to the practice testing string-shock absorber system, since an actual elastic pipe with limited length is considered in the model and the vibration response of any cross section of the perforated string can be obtained.

2) The influence of perforated string length, shock absorber stiffness coefficient, and shock absorber damping coefficient to PSSA system are investigated. The study found that the dynamic response of the PS system can be controlled in a reasonable range when the range of perforated string length, shock absorber stiffness coefficient, and shock absorber damping coefficient respectively are selected as 30～40 m, 180～210 KN/m, 5～15KN·s/m.

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REFERENCES
